

TOPICS : Error Analysis

- The density of a cube is measured by measuring its mass and the length of its side. If the maximum errors in the measurements of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is
 - 7%
 - 5%
 - 9%
 - 3%
- A physical quantity Q is found to depend on observables x , y and z , obeying relation $Q = \frac{x^3 y^2}{z}$.
 The percentage error in the measurements of x , y and z are 1%, 2% and 4% respectively. What is percentage error in the quantity Q ?
 - 11%
 - 4%
 - 1%
 - 3%
- The side of a cubical block when measured with a vernier callipers is 2.50 cm. The vernier constant is 0.01 cm. The maximum possible error in the area of the side of the block is
 - $\pm 0.01 \text{ cm}^2$
 - $\pm 0.02 \text{ cm}^2$
 - $\pm 0.05 \text{ cm}^2$
 - $\pm 0.10 \text{ cm}^2$
- A physical quantity is given by $X = M^a L^b T^c$. The percentage error in measurement of M , L and T are α , β and γ respectively. Then, the maximum % error in the quantity X is
 - $a\alpha + b\beta + c\gamma$
 - $a\alpha + b\beta - c\gamma$
 - $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$
 - None of these
- A certain body weighs 22.42 g and has a measured volume of 4.7cc. The possible error in the measurement of mass and volume are 0.01 g and 0.01 cc. Then maximum error in the density will be
 - 22%
 - 2%
 - 0.2%
 - 0.02%
- Which of the following is the most precise device for measuring length ?
 - A vernier callipers with 20 divisions on the vernier scale coinciding with 19 main scale divisions
 - A screw gauge of pitch 1 mm and 100 divisions on the circular scale
 - A spherometer of pitch 0.1 mm and 100 divisions on the circular scale
 - An optical instrument that can measure length to within a wavelength of light
- Percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in the estimation of kinetic energy obtained by measuring mass and speed will be
 - 8%
 - 2%
 - 12%
 - 10%
- The least count of the metre rod is 0.1 cm. What is the permissible error in the length of the rod measured with it ?
 - $\pm 0.2 \text{ cm}$
 - $\pm 0.1 \text{ cm}$
 - $\pm 0.05 \text{ cm}$
 - $\pm 0.01 \text{ cm}$
- In a side callipers, $(m + 1)$ number of vernier divisions is equal to m number of smallest main scale divisions. If d unit is the magnitude of the smallest main scale divisions, then the magnitude of the vernier constant is
 - $\frac{d}{(m+1)}$ unit
 - $\frac{d}{m}$ unit
 - $\frac{md}{(m+1)}$ unit
 - $\frac{(m+1)d}{m}$ unit
- The temperatures of two bodies measured by a thermometer are $t_1 = 20^\circ\text{C} \pm 0.5^\circ\text{C}$ and $t_2 = 50^\circ\text{C} \pm 0.5^\circ\text{C}$. The temperature difference and the error therein is
 - $30^\circ\text{C} \pm 1^\circ\text{C}$
 - $70^\circ\text{C} \pm 0.5^\circ\text{C}$
 - $30^\circ\text{C} \pm 0.5^\circ\text{C}$
 - $70^\circ\text{C} \pm 1^\circ\text{C}$

TOPICS : Error Analysis (SOLUTION)

1. (c) : $\therefore \rho = \frac{M}{L^3}$,
 $\therefore \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} = 3\% + 3(2\%) = 9\%$.

2. 8. (a) : $Q = \frac{x^3 y^2}{z}$
 The percentage error in the quantity Q is
 $\frac{\Delta Q}{Q} \times 100 = \left(3\frac{\Delta x}{x} + 2\frac{\Delta y}{y} + \frac{\Delta z}{z} \right) \times 100$
 $= 3\left(\frac{\Delta x}{x} \times 100\right) + 2\left(\frac{\Delta y}{y} \times 100\right) + \frac{\Delta z}{z} \times 100$
 $= 3 \times 1\% + 2 \times 2\% + 4\% = 11\%$

3. (c) : Here $l = 2.50$ cm and $\Delta l = 0.01$ cm
 Since $A = l^2 = (2.50 \text{ cm})^2$
 $\therefore \frac{\Delta A}{A} = 2 \cdot \frac{\Delta l}{l}$; $\frac{\Delta A}{A} = 2 \times \frac{0.01 \text{ cm}}{2.50 \text{ cm}}$
 $\therefore \Delta A = \frac{2 \times 0.01 \text{ cm}}{2.50 \text{ cm}} \times (2.50 \text{ cm})^2$
 $= 2 \times 0.01 \times 2.50 \text{ cm}^2 = 0.01 \times 5 \text{ cm}^2$
 or $\Delta A = \pm 0.05 \text{ cm}^2$

4. (a) : $X = M^a L^b T^c$
 Percentage error in X
 $\frac{\Delta X}{X} \times 100 = a\frac{\Delta M}{M} \times 100 + b\frac{\Delta L}{L} \times 100 + c\frac{\Delta T}{T} \times 100$ As given,
 $\frac{\Delta M}{M} \times 100 = \alpha$, $\frac{\Delta L}{L} \times 100 = \beta$, $\frac{\Delta T}{T} \times 100 = \gamma$
 \therefore Percentage error in X = $a\alpha + b\beta + c\gamma$.

5. (b) : Density $\rho = \frac{\text{mass } m}{\text{volume } V}$... (i)

Take logarithm on the both sides of eqn. (i), we get
 $\ln \rho = \ln m - \ln V$... (ii)

Differentiate eqn. (ii), on both sides, we get

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta V}{V}$$

Errors are always added for maximum error.

\therefore Maximum error in the density ρ will be

$$= \left[\frac{\Delta m}{m} + \frac{\Delta V}{V} \right] \times 100\% = \left[\frac{0.01}{22.42} + \frac{0.1}{4.7} \right] \times 100\% = 2\%$$

6. (d) : The most precise device is one whose least count is the least.

(a) Least count of vernier callipers

$$= 1 \text{ MSD} - 1 \text{ VSD} = 1 \text{ MSD} - \frac{19}{20} \text{ MSD}$$

$$= \frac{1}{20} \text{ MSD} = \frac{1}{20} \text{ mm} = \frac{1}{200} \text{ cm} = 0.005 \text{ cm}$$

(\because 1 MSD = 1 mm)

(b) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$$

7. (a) : As $K = \frac{1}{2}mv^2$

$$\therefore \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{2\Delta v}{v} \times 100 = 2\% + 2 \times 3\% = 8\%$$

8. (b) : Permissible error = \pm least count = ± 0.1 cm

9. 32. (a) : $(m+1)$ V.S.D. = m M.S.D.
 $1 \text{ V.S.D.} = \frac{m}{m+1} \text{ M.S.D.}$
 Vernier constant = $1 \text{ M.S.D.} - 1 \text{ V.S.D.}$
 $= 1 \text{ M.S.D.} - \left(\frac{m}{m+1} \right) \text{ M.S.D.}$
 $= \frac{1}{(m+1)} \text{ M.S.D.} = \frac{d}{m+1} \text{ unit}$

10. (a) : Here, $t_1 = 20^\circ\text{C} \pm 0.5^\circ\text{C}$
 $t_2 = 50^\circ\text{C} \pm 0.5^\circ\text{C}$
 The temperature difference of two bodies is
 $t = t_2 - t_1 = 50^\circ\text{C} - 20^\circ\text{C} = 30^\circ\text{C}$
 The error in temperature difference is given by
 $\Delta t = (\Delta t_1 + \Delta t_2)$
 $= (0.5^\circ\text{C} + 0.5^\circ\text{C}) = 1^\circ\text{C}$
 \therefore The temperature difference is $30^\circ\text{C} \pm 1^\circ\text{C}$.